Entrainment of Sand by Fluids

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Contents

Abstract

The entrainment of sand by fluids has been a large area of research in a variety of fields. The purpose of this study is to examine a few of the studies that have previously been performed and consequently apply them to a particular area, the Oregon Dunes National Recreation Area, which is a large eolian dune complex in south central coastal Oregon. After a literature search, three papers were decided on. One was dealing with entrainment by water, and the remaining dealt with entrainment by wind. The equation dealing with water was the most useful, although the field of most interest was the area of entrainment by wind. For entrainment by wind, one equation failed, and the other was too general. For entrainment by water, a grain diameter of 0.025cm and a bed roughness value of 0.0185 cm was used. The shore facing side of the dune was used, which often has an angle of about 20°. The results for this analysis are 34.03 cm s^{-1} or 7.61 mph. For entrainment by wind, the critical shear stress would be 23.02 cm s^{-1} or 5.15 mph. This result would be for a 0.025 cm quartz grain on a horizontal surface composed of equidimensional grains with a constant, laminar wind flow at a fixed height above the sand surface. Such a situation would be characteristic of a deflation plain.

Introduction

The entrainment of sand grains is one of the primary factors when it comes to the formation of eolian dunes, as well as many other situations where sediment transport is an important process. There are a number of fields that deal with the subject, such as hydraulic engineering, geomorphology, stratigraphy, civil engineering, and sedimentary geology. Various scientists have dealt with the issue of entrainment and the differing aspects of the process. Some of the first people to work on the subject were Shields (1936) and Bagnold (1941). There has been since a number of people that have worked on different aspects of the subject. For instance, Slingerland (1977) worked on the variables that influence the critical velocity of entrainment. Belly (1964) and Azizov (1977) worked with the effects of capillary action of water on the sand grains. Effects of soluble salts were studied by Gillette *et al*, (1980) and Nickling and Ecclestone (1981). Still others studied critical velocity on the basis of experimental data (Nickling, 1988)

The application of interest is the case of eolian dune genesis, activation, and reactivation. The equation sought for this study was one in which critical velocity could be found from the individual grain size, average bed grain size, and other field measurable variables. What was not desired was equations based on experimental data and lab-based coefficients. Unfortunately, many of the equations found were based on experimental data, or did not have the variables in them that were wanted. The equations that were carried out and checked were from Slingerland, 1977; Bagnold, 1941; and Dade *et al*, 1990. Each one dealt with the problem differently. Slingerland dealt with the entrainment of grains of different densities by water, and therefore placer deposits. Bagnold dealt with simple entrainment by wind, without taking into account many other variables. Dade *et al* dealt with the effects of biologic cohesion on sand grains. Others have dealt with specifics of the mechanics involved, such as turbulent bursting (Cao, 1997)

Description of the Beginning of Movement

Bagnold (1936) described the initiation of sand movement very well. In summary, he built a wind tunnel, observed, described, and photographed the initiation of movement in his wind tunnel.

As the wind speed was slowly turned up, there was no gradual entrainment, but rather a sudden change where the movement in the wind tunnel progressed downward until in reached a steady rate of flow. The beginning of flow was described visually. If the sand floor was pressed smooth, small craters about 1mm in diameter could be seen to appear suddenly. The craters would then multiply in numbers until the entire surface would appear like the typical finely irregular appearance of wind blown sand. The rise is nearly vertical and the fall is at a nearly constant downward angle of about 10°-16° from the horizontal. As the flow progressed, the craters coalesced laterally into ridges, which slowly combined and separated into the final ripple marks.

Finding Entrainment Values

Finding the critical velocities required for entrainment is could be as simple as plugging in a few numbers. However, to obtain a complete understanding of the movement of a grain, the physics of the system must also be understood.

The first step in understanding the initiation of movement is grasping what is going on the level of the individual grain in relation to the surrounding grains. This is gained through the knowledge of the forces acting on the grain by gravity, surrounding grains, and the force associated with the fluid movement. The variables involved are defined as the equilibrium requirements.

Equilibrium Requirements

The best way to gain an understanding of the forces in to generate a free body diagram of the interacting grains and forces. We must also make

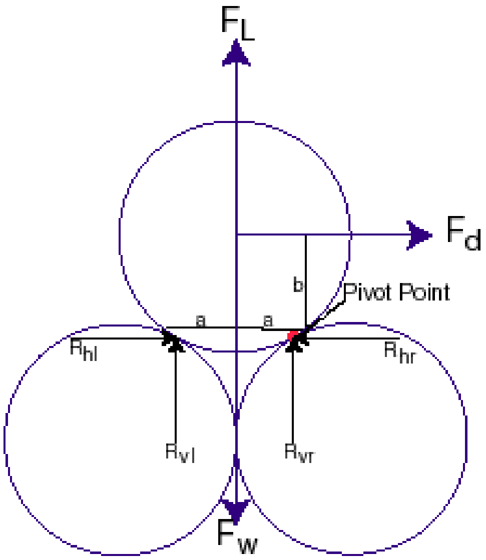
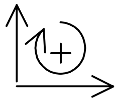


Figure 1 Free body diagram of grains and the related forces.

assumptions, or place limits on the system. The assumptions in this exercise are as follows: cohesionless, horizontal bed, contact points eqidistant from each other, and equidimensional grains. A free body diagram for this exercise is shown in figure 1.

The equations involved for the project are as put forward below, as well as the solution for the force requires to entrain the grain.

$$\left[\sum F_m = 0\right] \qquad \left[\sum F_x = 0\right] \qquad \left[\sum F_y = 0\right] \tag{1.}$$



$$\begin{aligned} \sum F_m &= aF_L + bF_D - aF_w + 2aR_{vl} = 0 \\ \sum F_x &= F_D - R_{hr} + R_{hl} \\ \sum F_y &= F_L + R_{vr} - F_w + R_{vl} \end{aligned} \tag{2.}$$

Solve for system of three equations

$$\begin{aligned} &\sum aF_L + bF_D - aF_w + 2aR_{vl} = 0 \\ &\sum F_D - R_{hr} + R_{hl} = 0 \\ &\sum F_L + R_{vr} - F_w + R_{vl} = 0 \end{aligned} \tag{3.}$$

Summation in the X direction can be ignored, since that both sides equal one another.

Solve for F_w in the Y direction: $F_w = F_l+R_{vr}+R_{vl}$

Plug into moment equilibrium eq.

Solution: $aF_l+bF_d-a(F_l+R_{vr}+R_{vl})+2aR_{vl} = 0$

Solve for F_d:

$$\begin{aligned} &1.)F_d = \frac{a}{b}\left(R_{vr} + R_{vl}\right) \quad ; R_{vr} + R_{vl} = F_{w,} \therefore \\ &2.)F_d = \frac{a}{b}F_w \end{aligned} \tag{4.}$$

Find parameters for a 1 mm diameter pure quartz spherical sand grain in air, in two dimensional cubic close packing with two other equidimensional sand grains lying on a flat surface. This gives the parameters of:

$F_w = \rho g V$

$\rho = 2.65\ \text{g/cm}^3$

Volume of grain = $\frac{4}{3}\pi r^3 = 5.2 \times 10^{-4}\ \text{cm}^3$

$g = 980\text{cm/s}^2$

a = 0.5 sin 30° = 0.25 mm

b = 0.5 sin 60° = 0.43 mm

Plug into equation:

$$F_d = \frac{0.25\text{cm}}{0.43\text{cm}}\left(2.65\ {}^{\text{g}}\!/\!_{\text{cm}^3} \times 980\ {}^{\text{cm}}\!/\!_{\text{s}^2} \times \left(5.2 \times 10^{-4}\ \text{cm}^3\right)\right)$$
$$F_d = 0.79\ {}^{\text{g}\cdot\text{cm}}\!/\!_{\text{s}^2} \qquad 5.$$
$$F_d = 7.9 \times 10^{-12}\ \text{N}$$

Therefore the force required to move the grain out equilibrium is 7.9×10^{-12} N.

Entrainment by Water, *From Slingerland, 1977:*

The next step in finding critical values was to understand how the sand grains are entrained by water. This is important since the majority of entrainment work is primarily concerned with water hydraulics, specifically channel erosion. The equation that I used for this part, since it has many of the important variables, was one from Slingerland, 1977. It is based on the Karmen-Prandtl equations for rough and smooth flow, and takes into account modifications by a variety of authors (Einstein, 1950; Leliavsky, 1966; Yalin, 1972; White, 1940; Miller and Byrne, 1966; and Ippen and Eagleson, 1955) and is as follows:

$$V_c = 5.75\log_{10}\left(\frac{30.2xy}{BKS}\right)\beta_1\beta_2\sqrt{\frac{4dg\cos\alpha(\rho_p - \rho_f)(\tan\phi - \tan\alpha)}{3C_d\rho_f}} \qquad 6.$$

Where

V_c = critical velocity for entrainment

d = grain diameter (mm)

BKS = roughness size = $d_{65\%}$[1]

Tanϕ = reactive force angle: $\tan\phi = \dfrac{0.866}{\left(\left(\dfrac{d}{BKS}\right)^2 + 2\left(\dfrac{d}{BKS}\right) - \frac{1}{3}\right)^{\frac{1}{2}}}$ 7.

ν = viscosity of the fluid

C_d =Coefficient of drag: $C_d = \dfrac{24}{Re}\left(1 + 0.15\,Re^{0.687}\right)$ 8.

Re = Critical Reynolds Number: $Re_c = \dfrac{V_c d}{\nu}$ 9.

x = Einstein correction factor (Einstein, 1950, Fig 4)
y = height of grain above datum on which it rests.
α = bed slope in degrees
g = gravitational acceleration
ρ_f = density of fluid
ρ_p = density of particle
β_1 = turbulent velocity fluctuation coefficient
β_2 = coefficient for point of application of fluid force

The terms β_1 and β_2 are defined as variables from the parameter U_*, the shear velocity, which is defined as:

$U_* = \sqrt{\dfrac{\tau_{xy}}{\rho}}$ (Simons and Sentürk, 1992) 10.

Where $\tau_{xy} = -\rho_f \overline{u'v'}$ where $\overline{u'v'}$ is the product of the average velocity fluctuations in the horizontal and vertical direction.

The solution for this equation matched Slingerland's solution for a 1mm grain, 42.73 cm s^{-1}. The solution was found by setting up iterations since that the critical velocity was both in the solution and in the equation. The complete problem is included in the appendix.

When attempting this equation for wind, the answer was not what was expected (1891 cm s^{-1}). The variables obviously are for entrainment by water only. I was

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[1] $d_{65\%}$ is defined as the grain size for which 65% of the distribution is finer.

disappointed by this and in that the equation had the variables that were sought after. It was suggested by the author that I not use this equation for wind.

The uses that I might have for this equation, in relation to the coastal dunes, is to find the critical velocities associated with overwash by waves and possible tsunamis. This application could be of use by determining grain sizes of tsunami deposits and then calculating the critical velocities needed to entrain them. Also of use would be equations for saltation to find the velocities of waves carrying in the sediment and calculating the velocity of the wave at the point of deposition.

Entrainment by Wind, *From Bagnold, 1941*

The simple forms of equations are also useful in understanding the basic components of entrainment. The following equation from Bagnold, 1941 is just such an equation.

This was the simplest equation to deal with, as there was no root finding, searching for definitions of variables, or iterations. The form of the equation is as follows:

$$u_{*t} = A\left(\frac{\rho_p - \rho_f}{\rho_f} \times g \times d\right)^{1/2} \qquad 11.$$

Where

A is a empirical coefficient equal to 0.1 for Reynold numbers >3.5 and with no saltation in progress (0.8 for saltation)

u_{*t} = critical shear velocity

d = grain diameter

The primary drawback to this equation is that it doesn't give the options of changing variables of the equation, such as the angles and other relationships of the primary grain in relation to the bed grains. The generalities of this equation would make it a quick way of estimating the critical wind speed required for entrainment of a single grain, and would do well in most cases.

In summary, the critical shear velocity to entrain a 1mm quartz grain by wind is 46 cm/s or 16.6 km/h. The complete results of this equation for a variety of grain sizes are given in the appendix.

Entrainment by Wind, *From Dade et al., 1990*

The equation for entrainment as presented in this work primarily dealt with the effects of biologic exopolymers on the entrainment of sand grains. On the outset of working with this equation, I was not sure whether this would provide me with the desired results. The equation was approached like the others, by first defining all the variables and then plugging them into the equation, which is as follows:

$$\rho_f u_{*cr}^2 = \frac{\rho_p - \rho_f}{d^2}\left(\frac{0.17\tan\Phi}{1+0.20(\mathrm{Re}_*)_{cr}\tan\Phi}\right)\left(\frac{1}{1+0.26(\mathrm{Re}_*)_{cr}}\right)\left(1+\frac{F_c}{\rho_p-\rho_f}\right) \qquad 12.$$

Where

Φ = a combined bed-packing and internal friction angle taken to be 65°

F_c = force of adhesion due to capillary action of water (originally taken to be the adhesive force due to the exopolymer *A. atlantica* exudate).

This equation was then changed around to be solved for u_{*cr} in the following manner:

$$u_{*cr} = \pm\sqrt{\frac{\frac{\rho_p - \rho_f}{d^2}\left(\frac{0.17\tan\Phi}{1+0.20(\mathrm{Re}_*)_{cr}\tan\Phi}\right)\left(\frac{1}{1+0.26(\mathrm{Re}_*)_{cr}}\right)\left(1+\frac{F_c}{\rho_p-\rho_f}\right)}{\rho_f}} \qquad 13.$$

The misgivings present were that the coefficients, such as 0.17, 0.20, and 0.26 might have been designed for water only, and that they would fail when attempting it for air.

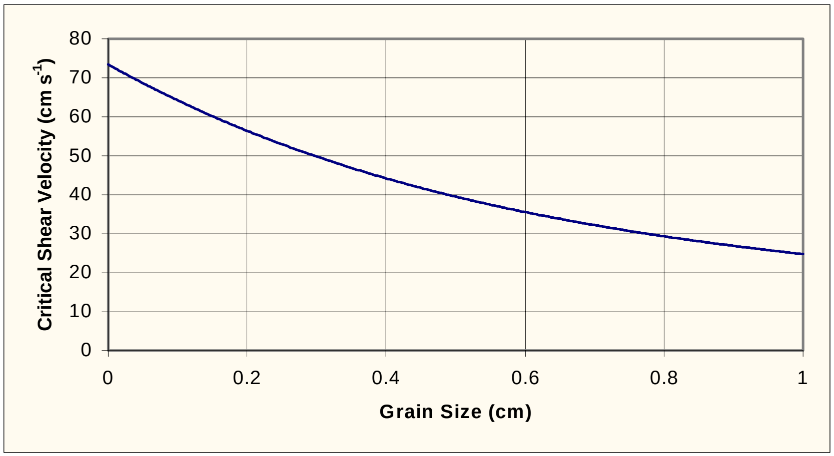

Figure 2 Chart showing critical velocity versus grain size. Negative trending curve indicates a relationship of lower entrainment velocities with increasing grain sizes, which is incorrect.

Also, as with the other equations, the term of u_* was on both sides of the equation (in the form of Re_* which equals $\left(\frac{u_*}{d}\right)\upsilon$).

After solving this equation, the relationship of critical velocity versus grain size was graphed. A unexpected trend was the result (see figure 2). This relation made the equation, as presented in the text, incorrect.

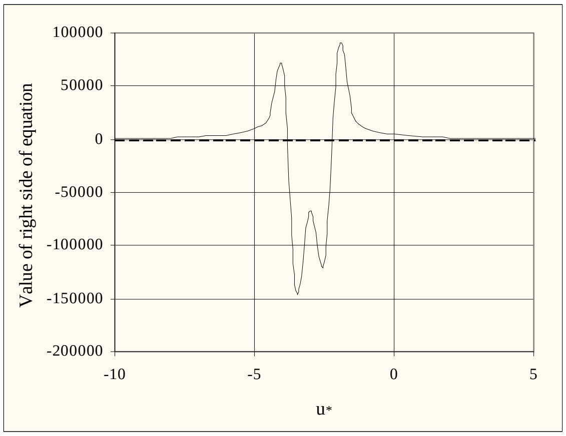

Figure 3 Curve showing the relation between u_* and the right side of equation 12.

As a double-check to this problem, the root finding method of bisection was employed to check the equation. The solutions for various grain sizes displayed the same trends. A curve was generated to see if there were multiple roots to the right side of equation 12. Figure 3 shows the resulting relationship. While it seems that the equation assumes no negative velocities, for values of u_* between zero and negative five the result fluctuates wildly between 100,000 and -150,000.

In the article, the authors said that the equation was in development, the equation could have changed since publication. The authors of the equation have been contacted, but there has no response. It is possible that the equation is valid for only particular parameters, such as one grain size or temperature. It is possible that the given coefficients in the equation did not work out for air, although they were also tried for water and the same negative trends appeared.

For purposes of this project, this equation failed.

Application of Equations to my Area of Study

The area that I have been working in is the Oregon Dunes National Recreation Area (ODNRA). This is the largest dune complex on the West Coast at about 70km running north south. An extensive grain size analysis of the dune sands, and their corresponding soil horizons (Peterson and Stock, 1997) has been performed. I took an average sample from the analysis (number 116, see appendix) and then proceeded to perform an analysis for water using Slingerland (eq. 6), and one for wind using Bagnold's equation (eq. 11)

For entrainment by water, I used a grain diameter of 0.025 cm and the $d_{65\%}$ value was 0.0185cm. I will use the stoss side of the dune, which often has an angle of about 20°. The results for this analysis are 34.03 cm s^{-1} or 7.61 mph (see Appendix). Therefore, a non-bedload bearing tsunami, or any other wave, would only have to be moving at 7.61 mph up a 20° slope to pick up a 0.025 cm quartz sand grain on a average bed roughness of 0.0185 cm.

For wind to do the same thing within the limitations of Bagnold's equation (i.e. no bed roughness; slope; or other physical variables) the critical shear velocity would be 23.02 cm s^{-1} or 5.15 mph. This result would be for a 0.025 cm quartz grain on a horizontal surface composed of equidimensional grains with a constant, laminar wind flow at a fixed height above the sand surface. This simple type of model would work well for deflation plains in the lee side of the dune. The low critical velocity also holds to the idea that the deflation plains exhibit high average grain sizes when compared to the dunes.

Conclusions

The limitations of Bagnold's equation seem clear. There would need to be a more inclusive equation for wind, such as Slingerland's equation is for water. Unfortunately, the works that might have what is needed are obscure, often governmental technical reports, university technical reports, or foreign scientific journals, none of which are carried in local libraries. For further research in the subject, I would need to obtain some of these articles and continue to work through the problems.

As for the objective of this project, I am not satisfied. I still am not able to say, from a non-laboratory perspective, what the critical velocities for entrainment of sand by air are. A simple, very restricted value is obtained (via Bagnold), which in most cases would be adequate. Still, a more encompassing equation is needed for a complete quantification. Such an equation would take into account all the variables of Slingerland, plus such additional variables such as turbulent bursting and cohesion by water, soluble salts, biologic activity, or electrostatic charges. It was hoped that the work by Dade *et al* would have helped with this, but it failed. There are other works out there, but they are in the aforementioned obscure journals and reports.

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Appendix

Basic Solutions

1. Critical Velocity for Sand Entrainment by Water; from Slingerland, 1977
2. Critical Velocity for Sand Entrainment by Wind; from Bagnold, 1941
3. Critical Velocity for Sand Entrainment by Wind; from Dade *et al*, 1990

Applied Solutions

4. Sieve Analysis for ODNRA Sample 116
5. Critical Velocity for Sand Entrainment by Water; from Slingerland, 1977